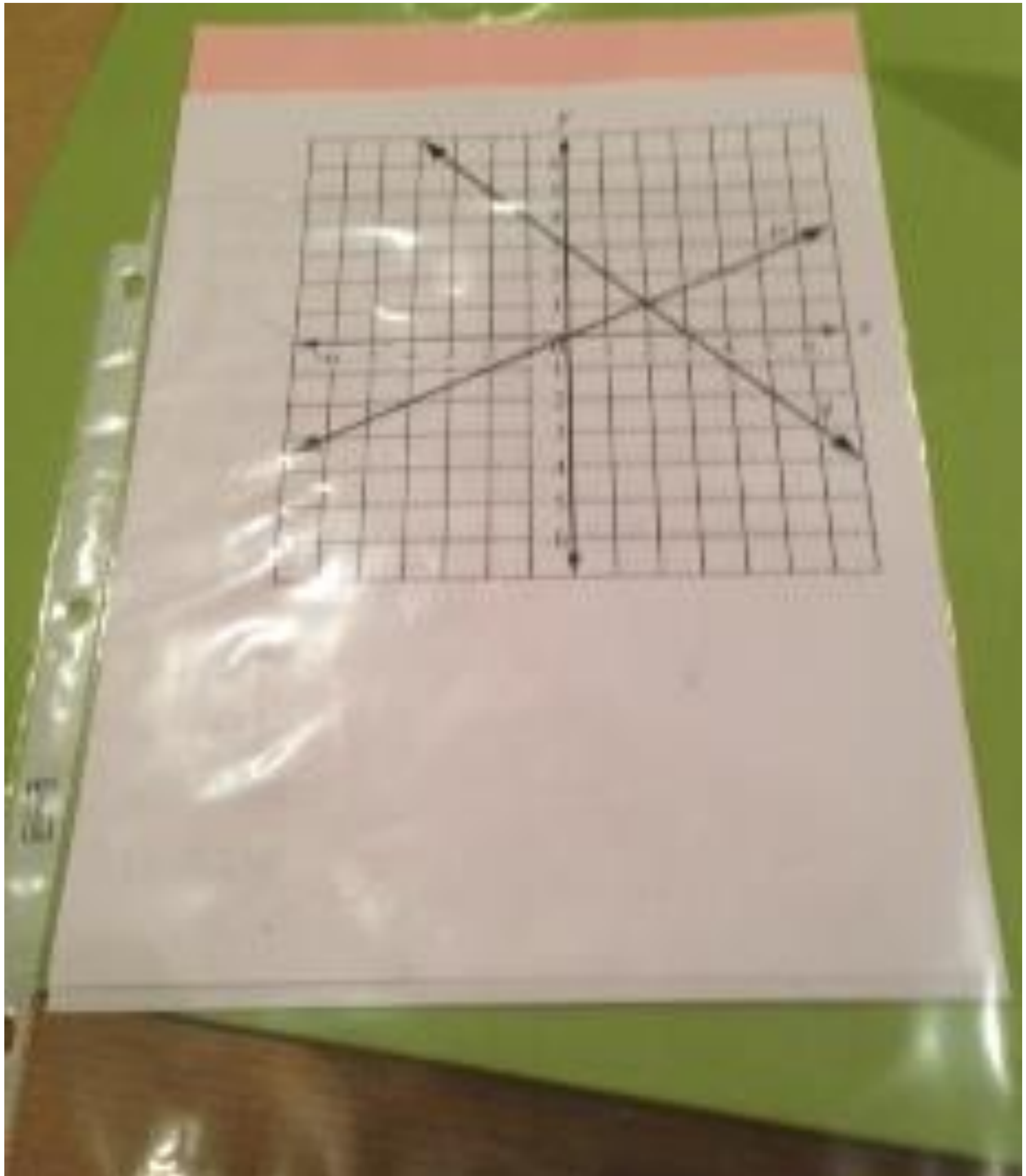
The One Penny Whiteboard

Ongoing, “in the moment” assessments may be the most powerful tool teachers have for improving student performance. For students to get better at anything, they need lots of quick rigorous practice, spaced over time, with immediate feedback. The One Penny Whiteboards can do just that.

To add the One Penny White Board to your teaching repertoire, just purchase some sheet protectors and white board markers (see the following slides). Next, find something that will erase the whiteboards (tissues, napkins, socks, or felt). Finally, fill each sheet protector (or have students do it) with 1 or 2 sheets of card stock paper to give it more weight and stability.



y
x
p
q

Expo Low Odor Chisel Tip Dry Erase Markers, 12 Black Markers (80001) by Expo

~~$21.99~~ **$8.39** Prime

In Stock

More Buying Choices

$8.29 new (55 offers)

$8.58 used (1 offer)

(120)

FREE Shipping on orders over $35

Product Features

Pack of 12 *Markers*

Office Products: See all 63 items

Expo Low Odor Fine Tip Dry Erase Markers, 12 Black Markers (86001) by Expo

~~$16.99~~ **$7.61** Prime

Order in the next **22 hours** and get it by **Thursday, Jan 9**.

More Buying Choices

$7.61 new (38 offers)

(54)

FREE Shipping on orders over $35

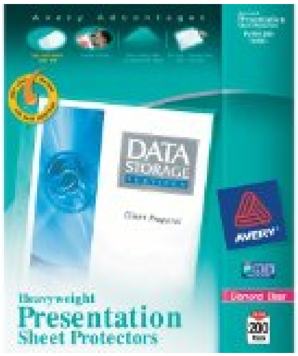
Product Features

... Low odor *dry erase marker* with great erasability. Nontoxic ink works ...

Office Products: See all 63 items

On Amazon, markers can be found as low as $0.63 each. (That's not even a bulk discount. Consider "low odor" for students who are sensitive to smells.)

I like the heavy-weight model.



Avery Diamond Clear Heavyweight Sheet Protector 200 Pack (74400) by Avery (Jan 21, 2009)

~~$42.99~~ **$18.99** Prime

Order in the next **20 hours** and get it by **Thursday, Jan 9**.

More Buying Choices

$16.95 new (37 offers)

$17.00 used (1 offer)

(138)

FREE Shipping on orders over $35

Product Features

Includes 200 *sheet protectors*

Office Products: See all 10,978 items

Avery Top Loading Clear Sheet Protectors, Heavyweight, 250 per Box #76006 by Avery

$21.39 Prime

In Stock

More Buying Choices

$17.71 new (16 offers)

$19.16 used (5 offers)

(47)

FREE Shipping on orders over $35

Product Features

Includes 250 *sheet protectors*

Office Products: See all 10,978 items

On Amazon, Avery protectors can be found as low as $0.09 each.

One Penny Whiteboards and The Templates

The One Penny Whiteboards have advantages over traditional whiteboards because they are light, portable, and able to contain a template. (A template is any paper you slide into the sheet protector). Students find templates helpful because they can work on top of the image (number line, graph paper, hundreds chart…) without having to draw it first. For more templates go to www.collinsed.com/billatwood.htm)

Using the One Penny Whiteboards

There are many ways to use these whiteboards. One way is to pose a question, and then let the students work on them for a bit. Then say, "Check your neighbor's answer, fix if necessary, then hold them up." This gets more students involved and allows for more eyes and feedback on the work.

Using the One Penny Whiteboards Group Game

One way to use the whiteboards is to pose a challenge and make the session into a kind of game with a scoring system. For example, make each question worth 5 possible points.

Everyone gets it right: 5 points
Most everyone (4 fifths): 4 points
More than half (3 fifths): 3 points
Slightly less than half (2 fifths): 2 points
A small number of students (1 fifth): 1 point

Challenge your class to get to 50 points. Remember students should check their neighbor’s work before holding up the whiteboard. This way it is cooperative and competitive.

Using the One Penny Whiteboards Without Partners

Another way to use the whiteboards is for students to work on their own. Then, when students hold up the boards, use a class list to keep track who is struggling. After you can follow up later with individualized instruction.

Keep the Pace Brisk and Celebrate Mistakes

However you decide to use the One Penny Whiteboards, keep it moving! You don't have to wait for everyone to complete a perfect answer. Have students work with the problem a bit, check it, and even if a couple kids are still working, give another question. They will work more quickly with a second chance. Anytime there is an issue, clarify and then pose another similar problem.

Celebrate mistakes. Without them, there is no learning. Hold up mistakes and say, "Now, here is an excellent mistake–one we can all learn from. What mistake is this? Why is this tricky? How do we fix it?"

The Questions Are Everything!

The questions you ask are critical. Without rigorous questions, there will be no rigorous practice or thinking. On the other hand, if the questions are too hard, students will be frustrated. They key is to jump back and forth from less rigor to more rigor. Also, use the models written by students who have the correct answer to show others. Once one person gets it, they all can get it.

Questions

When posing questions for the One Penny Whiteboard, keep several things in mind:

1. Mix low and high level questions
2. Mix the strands (it may be possible to ask about fractions, geometry, and measurement on the same template)
3. Mix in math and academic vocabulary (*Calculate* the **area**… use an **expression**… *determine* the *approximate* **difference**)
4. Mix verbal and written questions (project the written questions onto a screen to build reading skills)
5. Consider how much ink the answer will require and how much time it will take a student to answer (You don't want to waste valuable ink and you want to keep things moving.)
6. To increase rigor you can: work backwards, use variables, ask "what if", make multi-step problems, analyze a mistake, ask for another method, or ask students to briefly show why it works

Each of these questions can be solved on the One Penny Whiteboard.

To mix things up, you can have students "chant" out answers in choral fashion for some rapid fire questions. You can also have students hold up fingers to show which answer is correct.

Remember, to ask verbal follow-ups to individual students: Why does that rule work? How do you know you are right? Is there another way? Why is this wrong?

Examples

What follows are some sample questions that address: Algebra Common Core Standards ASSE 2-3, A-APR 3

This is also an introduction to the box method and applications of the distributive property with whole numbers in order to connect the math to prior learning.

There are also several slides to help struggling students learn the square numbers from 11-18. Hopefully this will connect to the box method for factoring and later with completing the square.

CONCEPTUAL CATEGORY: Algebra [A]

Content Standards

Seeing Structure in Expressions A-SSE

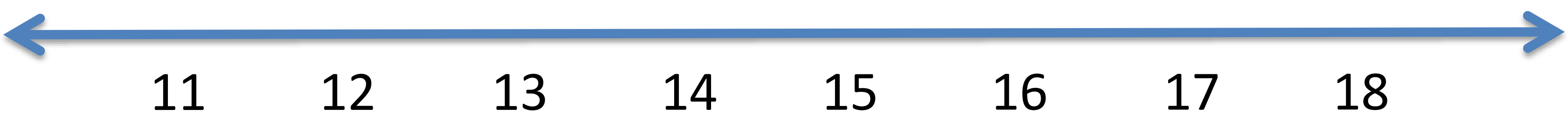
Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context. *
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.*
2. Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.*

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
   a. Factor a quadratic expression to reveal the zeros of the function it defines.
   b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
   c. Use the properties of exponents to transform expressions for exponential functions. *For example, the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.* *

Teachers: Print the following slides (as needed) and then have students insert whichever one you need into their whiteboards.


11
12
13
14
15
16
17
18

THE BOX METHOD

A WAY TO DISTRIBUTE WHEN MULTIPLYING A TWO DIGIT NUMBER

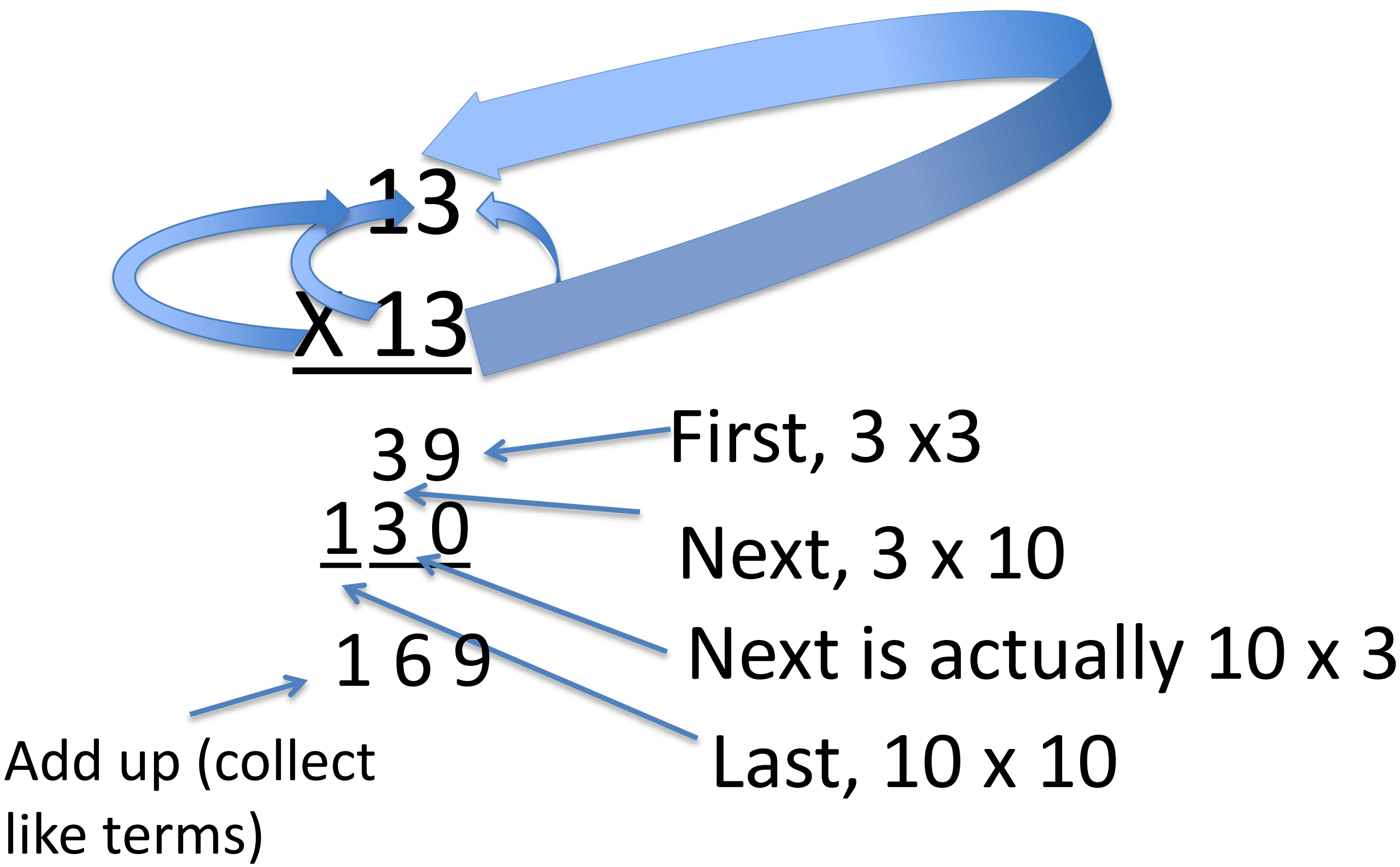
$13 \text{ X } 13 = \quad (10 + 3)(10 + 3)$

THE BOX METHOD

ALSO A WAY TO DISTRIBUTE WHEN MULTIPLYING TWO BINOMINALS.

(X + 3) (X + 3)

You've actually been doing this since fourth grade. Remember this?

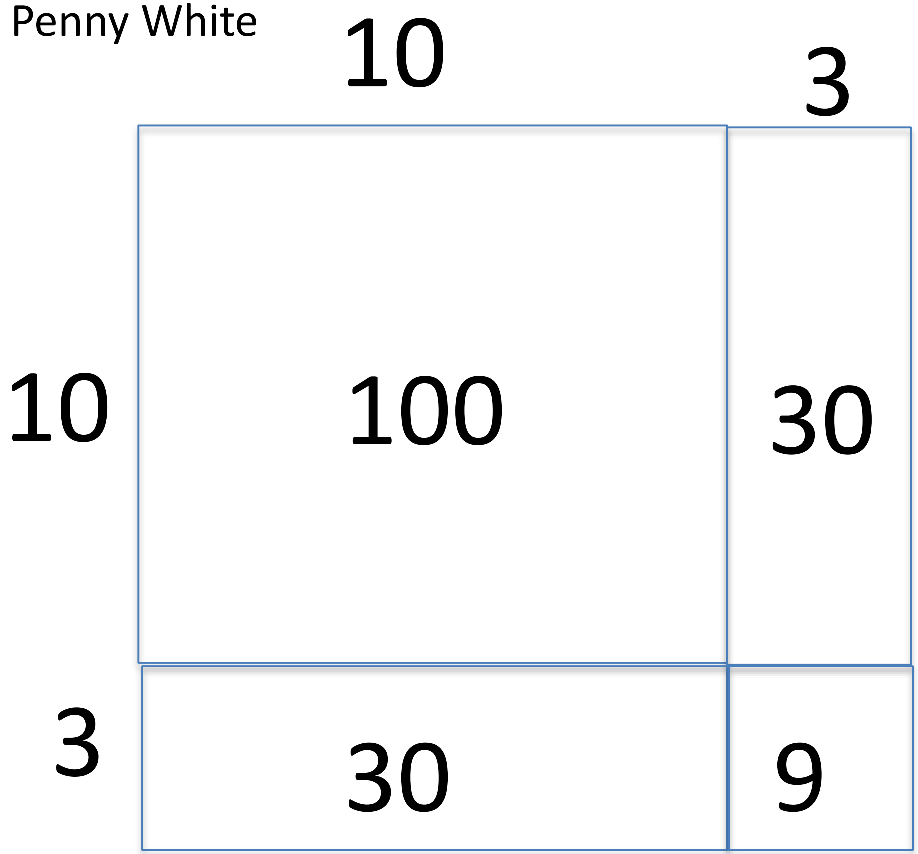


You can think about 13 X 13 as 4 sub-products.

One way to see this and do a lot of multiplying in your head is to picture multiplying as the area of a rectangle. Remember length x width…

Think of 13 x 13 as a (10 + 3) by (10 + 3) rectangle. Use your white board. (see slide 21)

Insert this template into your One Penny White Board.



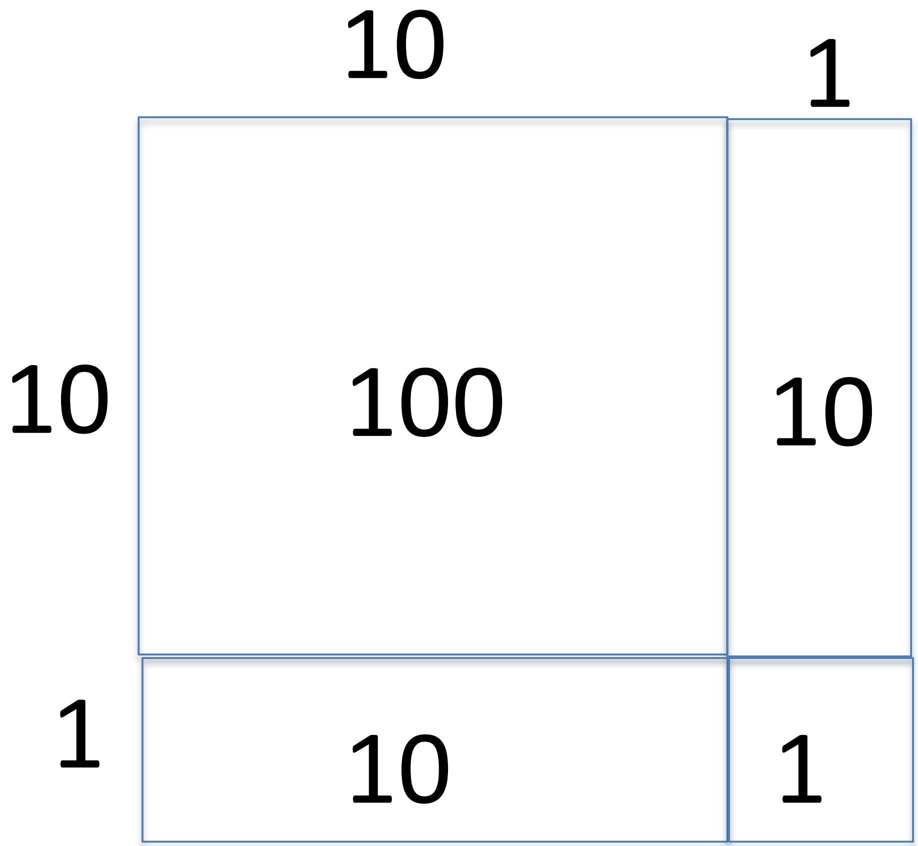
$$100 + 30 + 30 + 9$$

$$160 + 9 = 169$$

$$13^2 = 169$$

$$13^2 = (13)\,(13)$$

$$(10 + 3)\,(10 + 3)$$



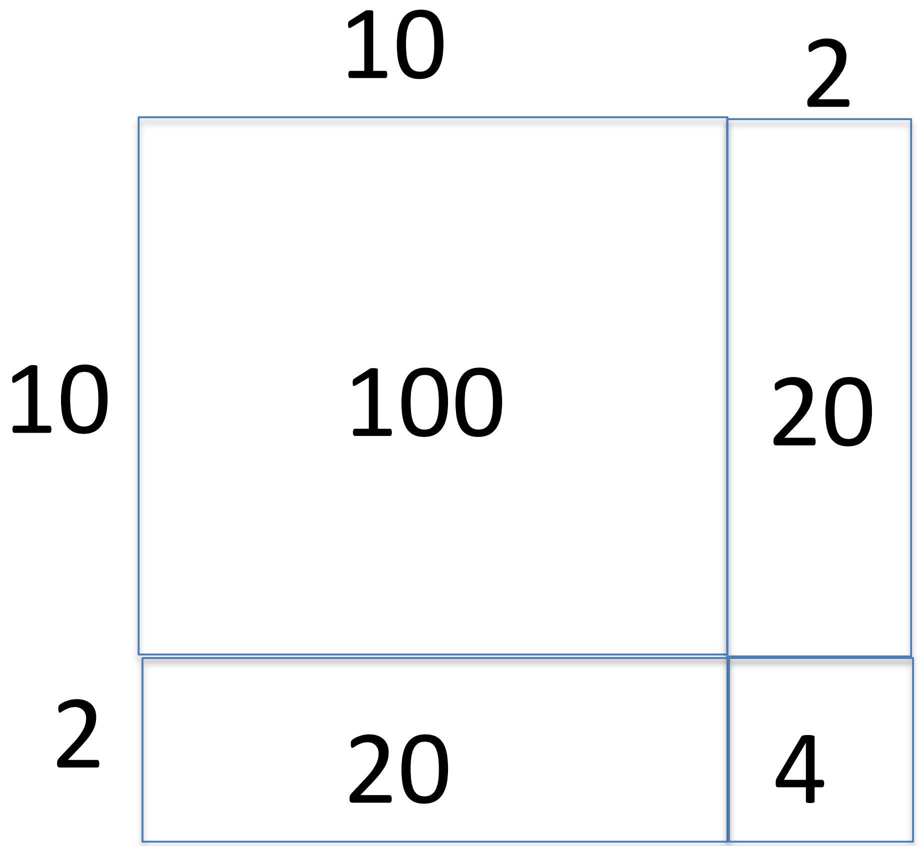
$$100 + 10 + 10 + 1$$

$$120 + 1 = 121$$

$$11^2 = 121$$

$$11^2 = (11)\,(11)$$

$$(10 + 1)\,(10 + 1)$$



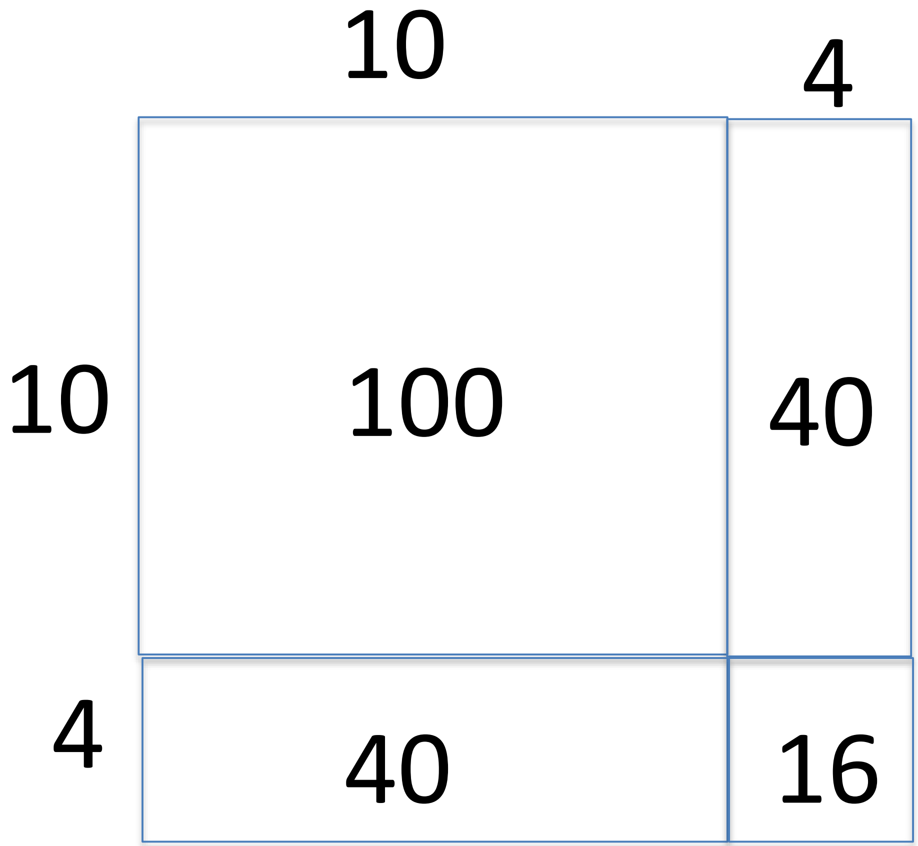
$$100 + 20 + 20 + 4$$

$$140 + 4 = 144$$

$$12^2 = 144$$

$$12^2 = (12)\,(12)$$

$$(10 + 2)\,(10 + 2)$$



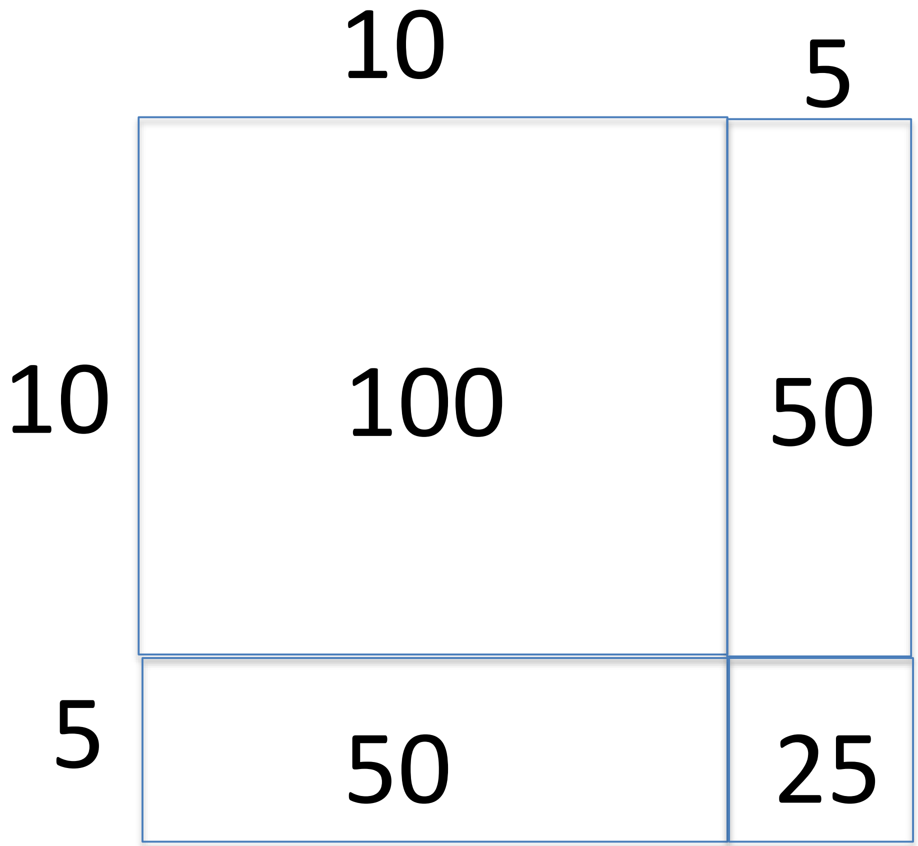
$100 + 40 + 40 + 16$

$180 + 16 = 196$

$14^2 = 196$

$14^2 = (14)\,(14)$

$(10 + 4)\,(10 + 4)$



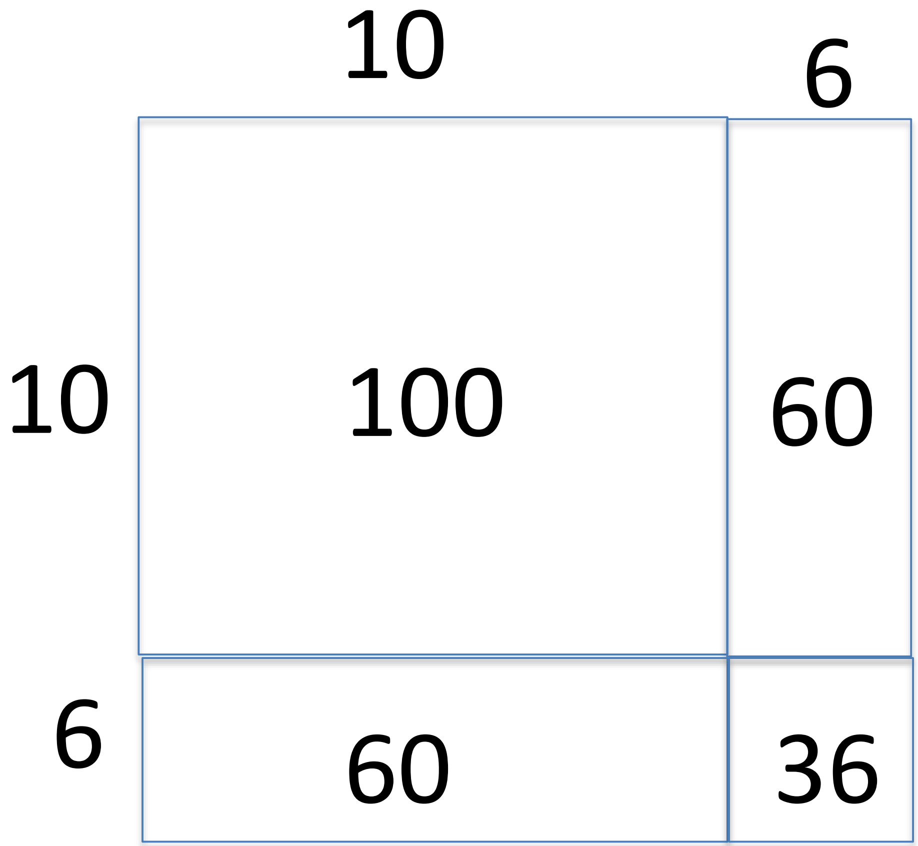
$$100 + 50 + 50 + 25$$

$$200 + 25 = 225$$

$$15^2 = 225$$

$$15^2 = (15)\,(15)$$

$$(10 + 5)\,(10 + 5)$$



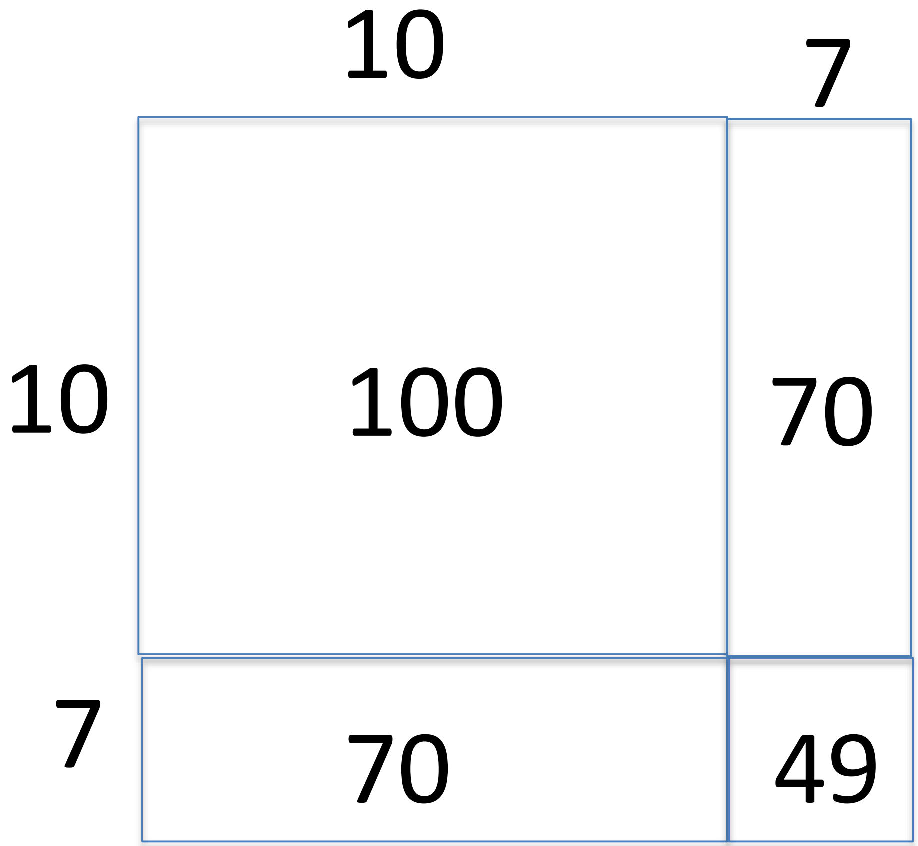
$$100 + 60 + 60 + 36$$

$$100 + 120 + 36 = 256$$

$$16^2 = 256$$

$$16^2 = (16)\,(16)$$

$$(10 + 6)\,(10 + 6)$$



$100 + 70 + 70 + 49$

$100 + 140 + 49 =$

$240 + 49 = 289$

$17^2 = 289$

$17^2 = (17)\,(17)$

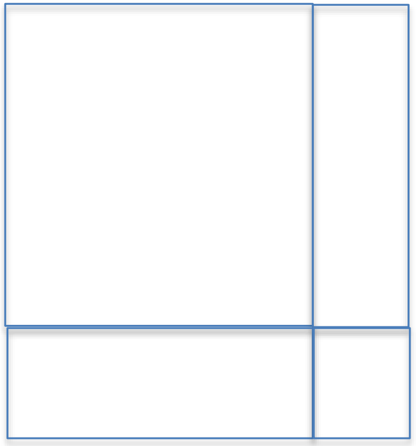
$(10 + 7)\,(10 + 7)$

Now you can visualize these as area problems, you can probably solve these more easily. Plus it will help you later to understand “completing the square” which leads to the quadratic formula.

Write the product on
your white board.

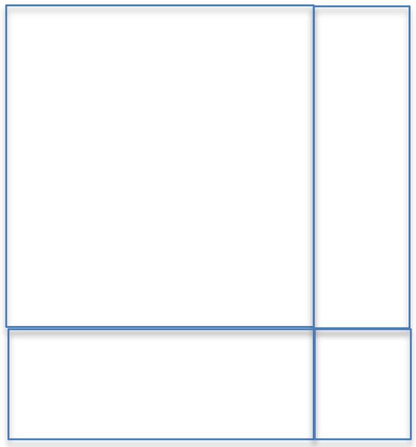
11^2

$11^2 = 121$



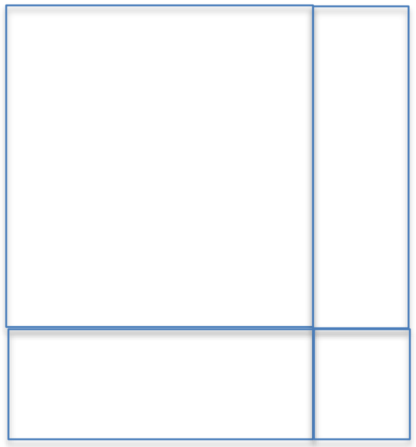
12^2

$12^2 = 144$



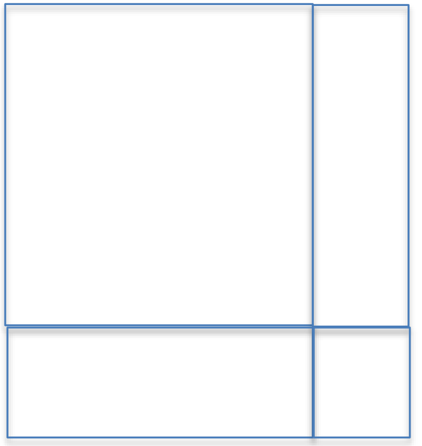
13^2

$14^2 = 169$



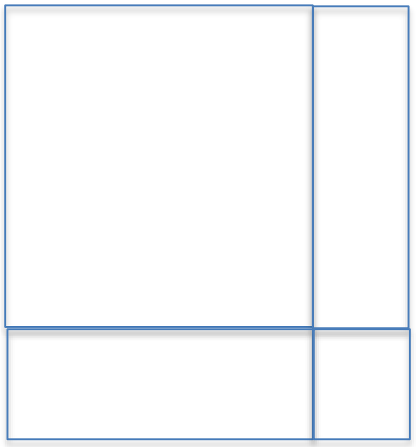
14^2

$14^2 = 196$



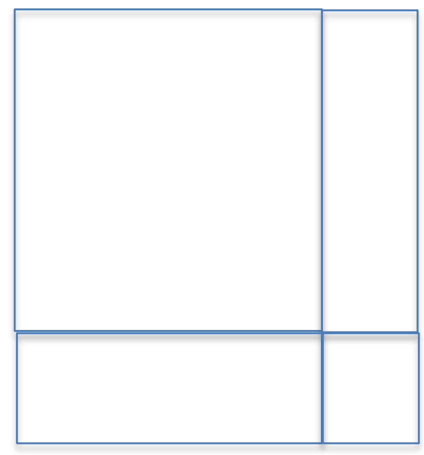
15^2

$15^2 = 225$



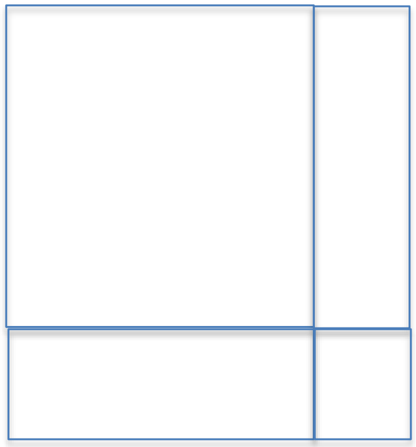
16^2

$16^2 = 256$



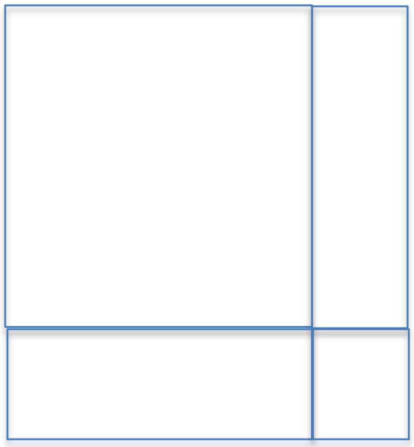
17^2

$17^2 = 289$



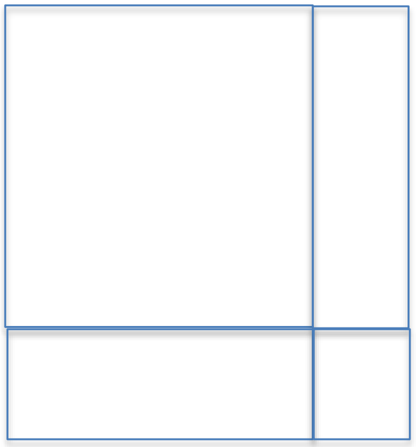
18^2

$18^2 = 324$



12^2

$12^2 = 144$

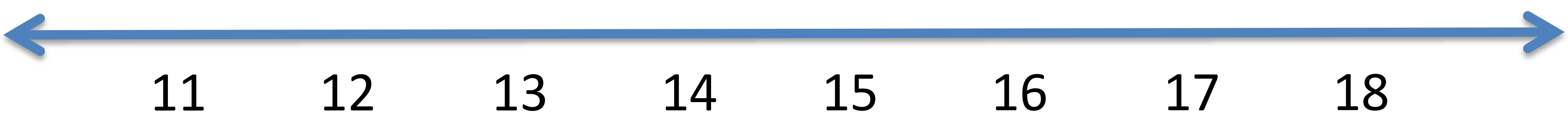


You need this information when you are asked to approximate square roots.

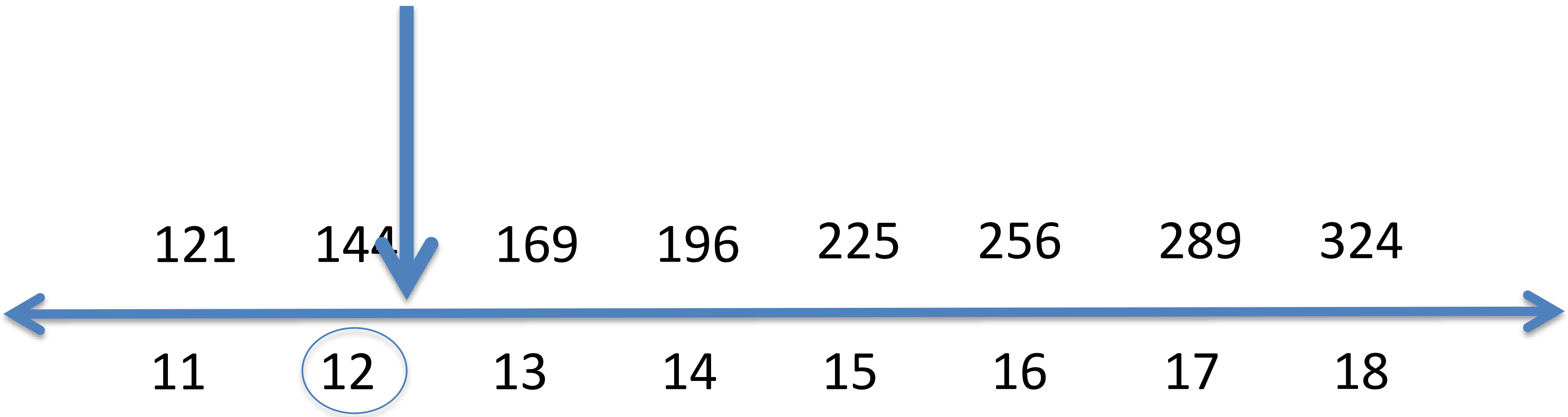
You know the square root of 144 is 12 because (12)(12) = 144

But what is the square root of 150?

Insert this template into your One Penny White Board.

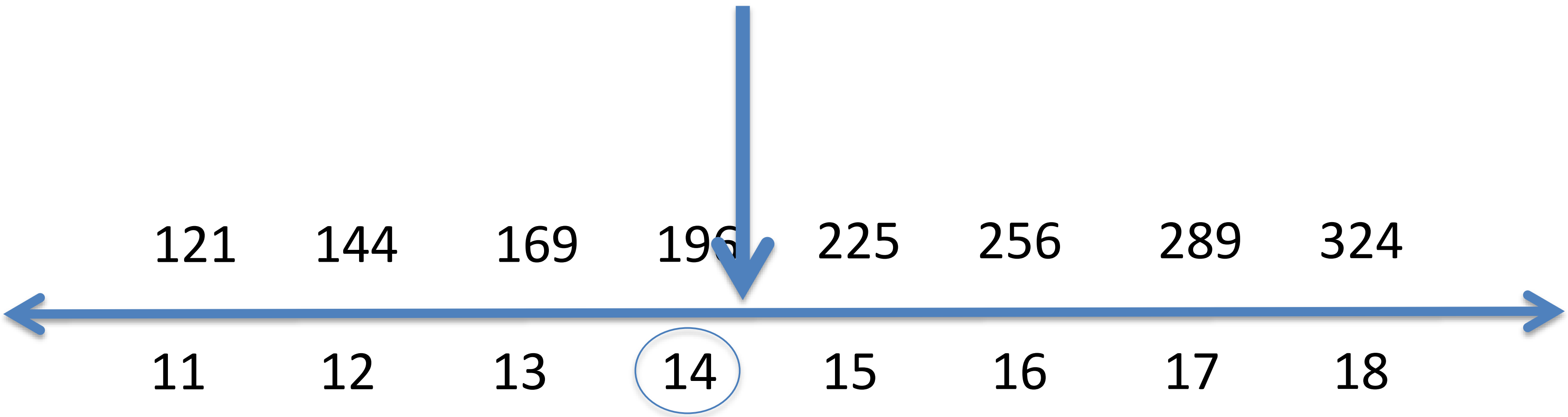


The square root of 150 is closest to which whole number?



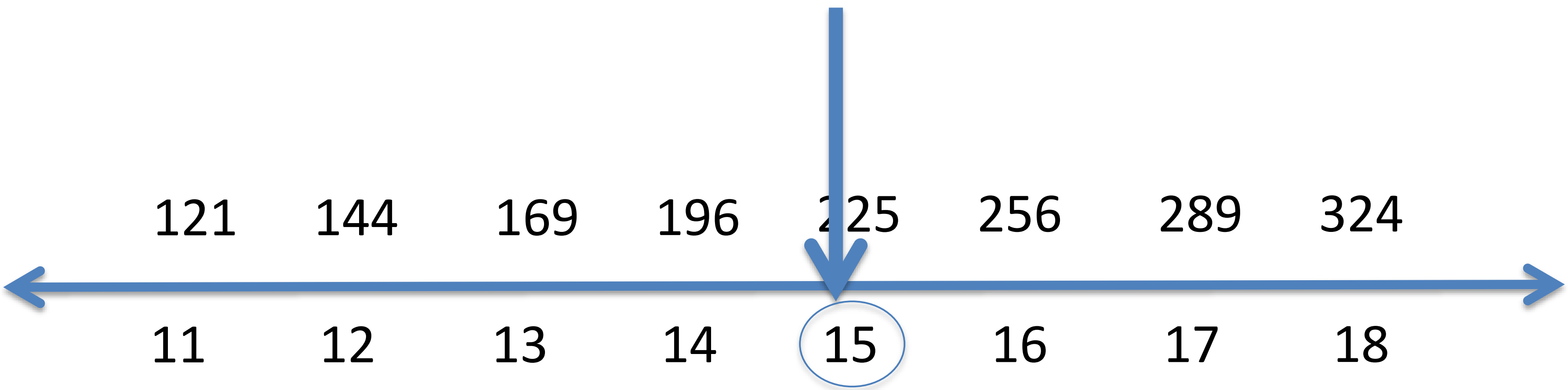
Think! And write the squares for each

The square root of 200 is closest to which whole number?



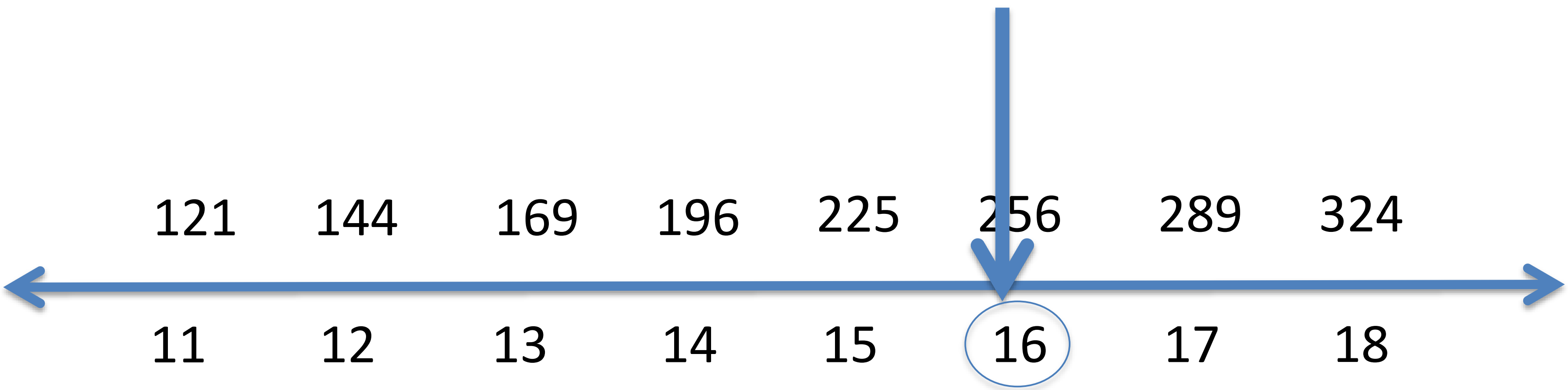
Think! And write the squares for each

The square root of 220 is closest to which whole number?



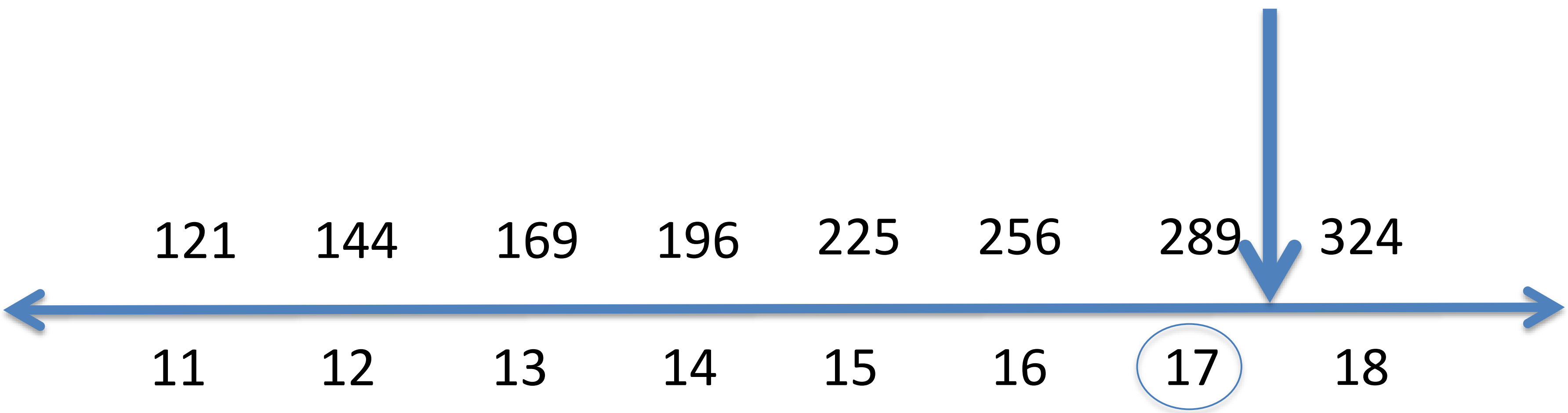
Think! And write the squares for each

The square root of 250 is closest to which whole number?



Think! And write the squares for each

The square root of 300 is closest to which whole number?

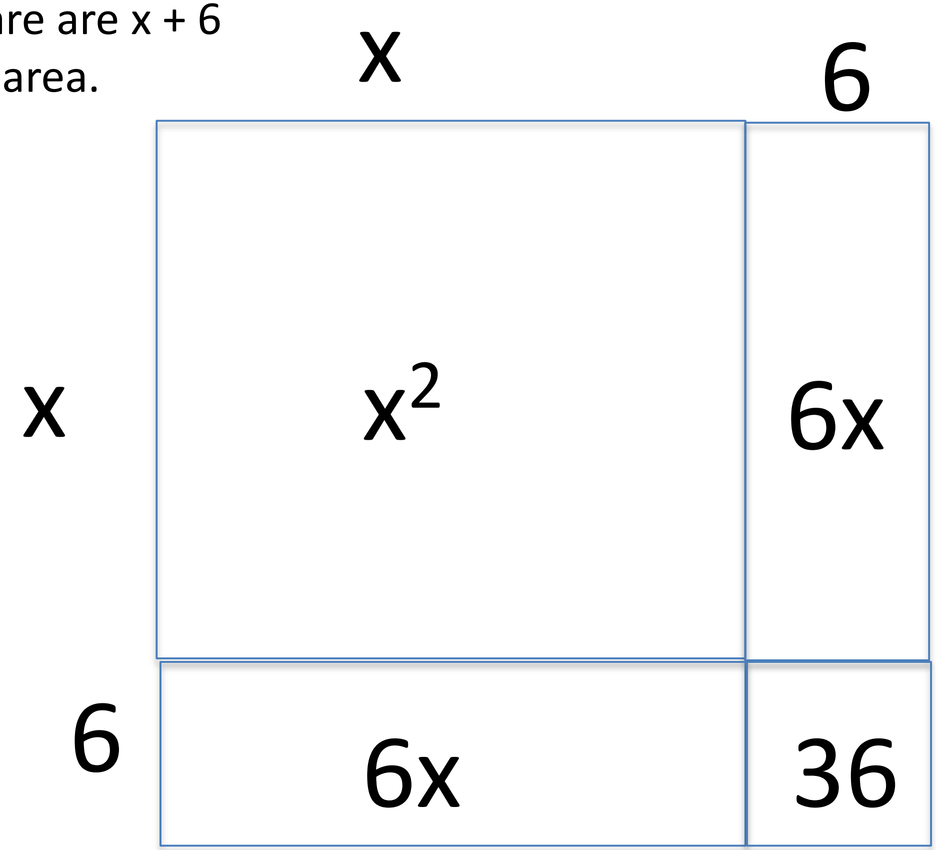


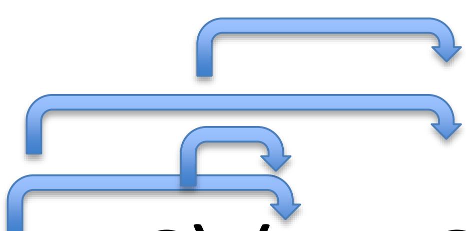
Think! And write the squares for each

Now, let’s practice the box method with some binomials that have variables. Put them in the box and distribute. You can see how it all connects.

Insert this template into your One Penny White Board.

Imagine the lengths of
this square are x + 6
Find the area.

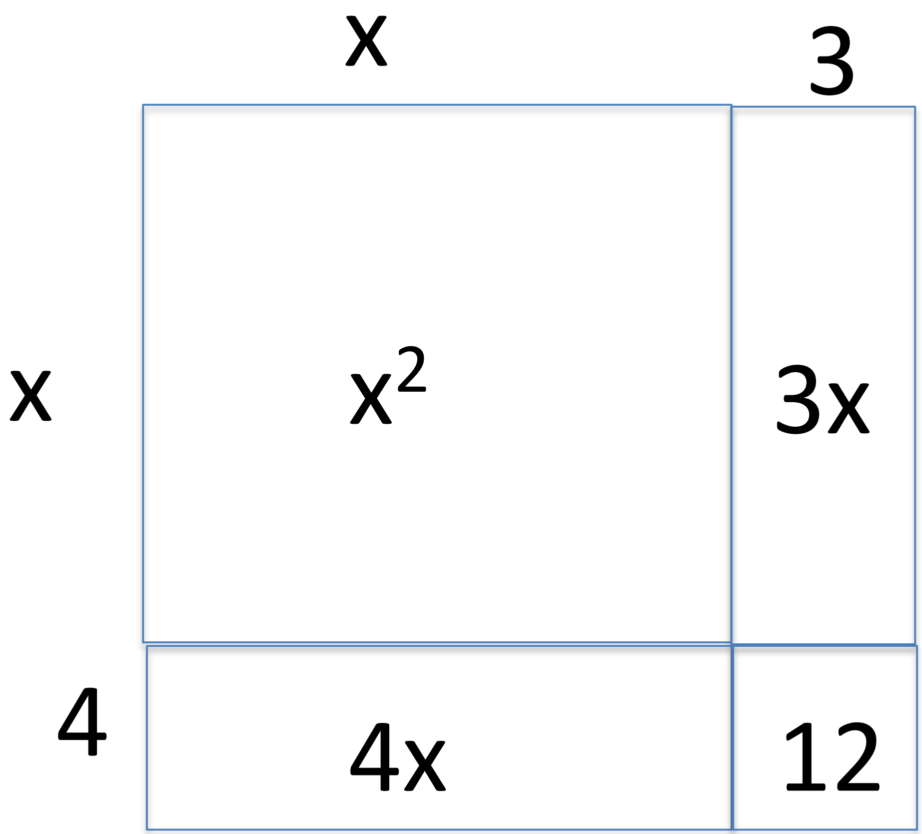


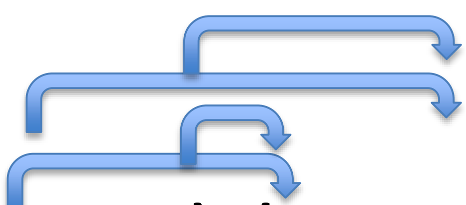


$(x + 6)\ (x + 6)$

$X^2 + 6x + 6x + 36$

$X^2 + 12x + 12$

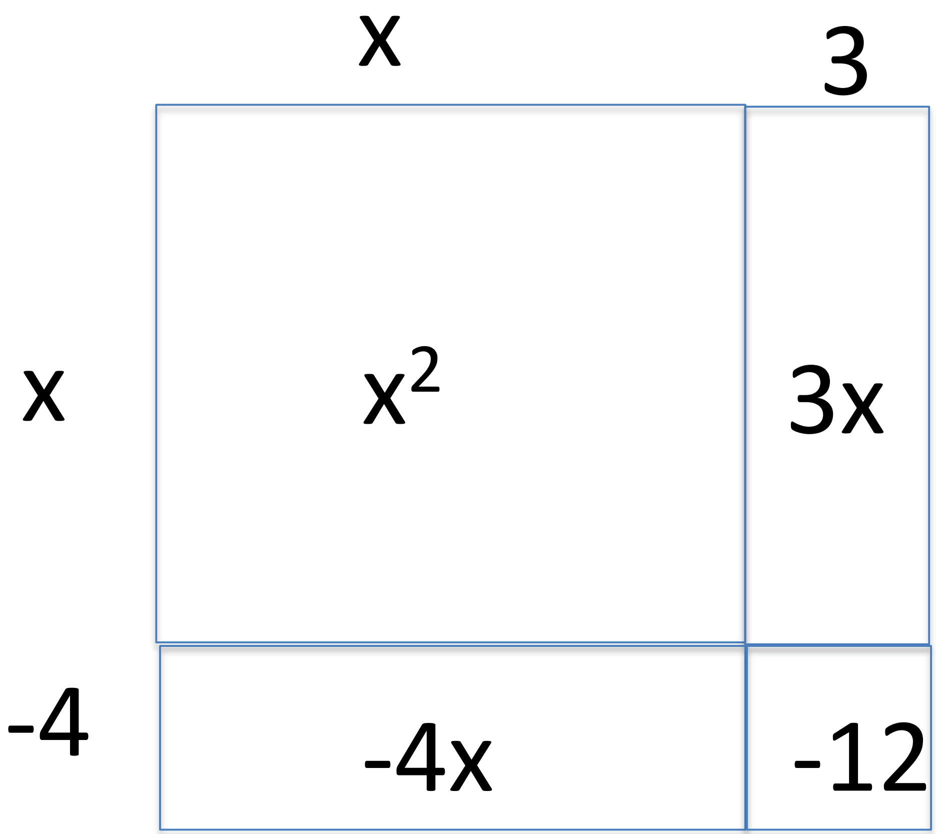


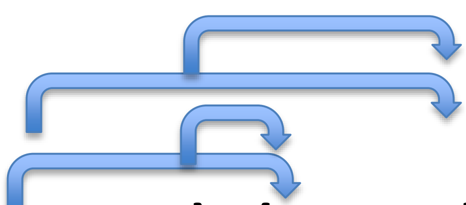


$(x + 3)\ (x + 4)$

$X^2 + 3x + 4x + 12$

$X^2 + 7x + 12$

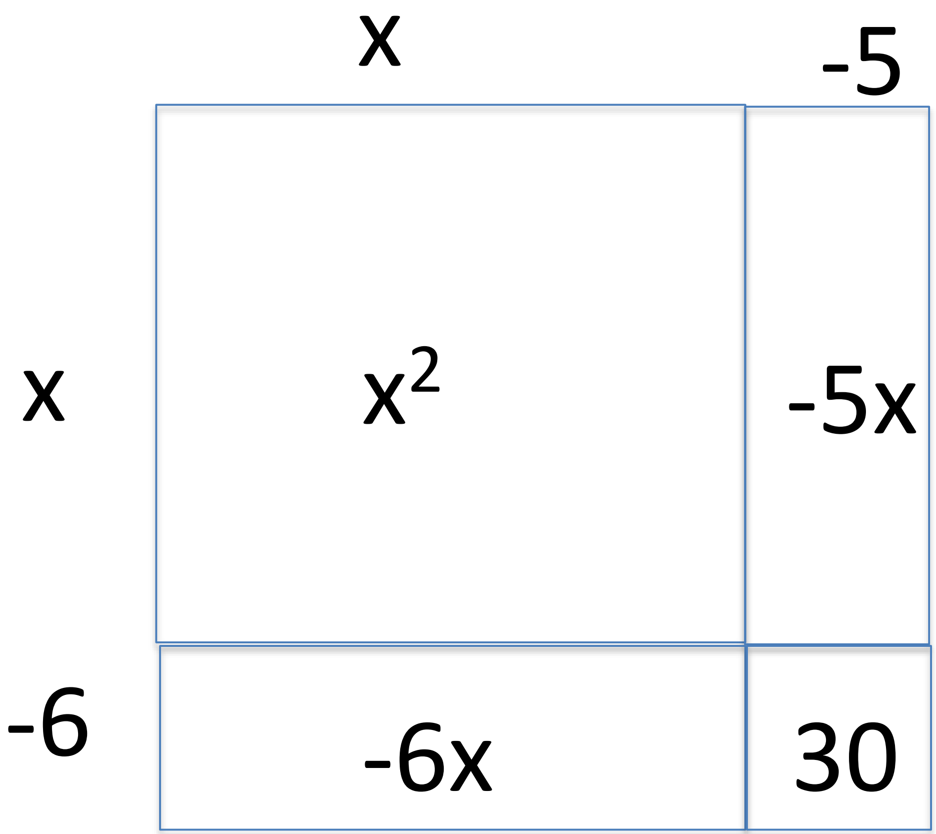


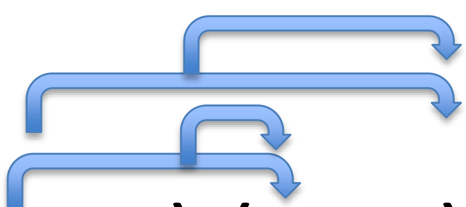


$(x + 3)\ (x - 4)$

$X^2 + 3x - 4x - 12$

$X^2 + -x + 12$

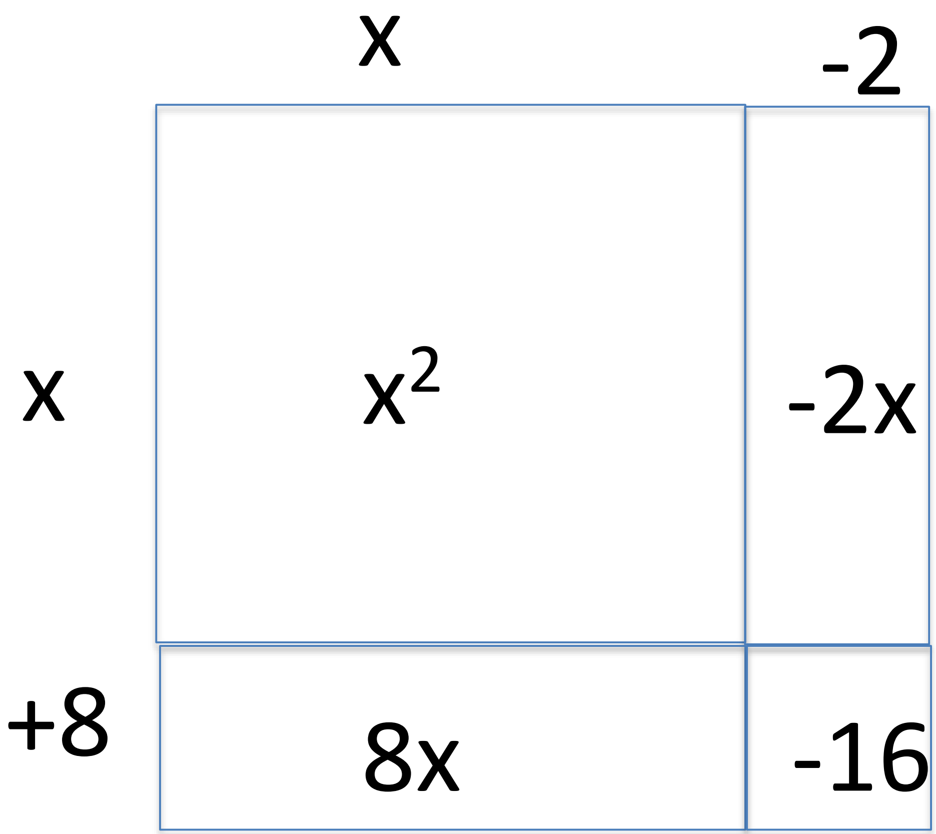


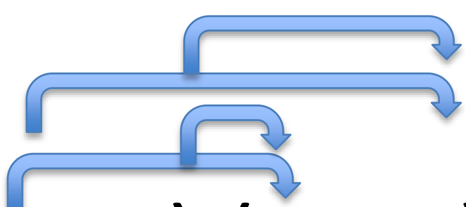


$(x - 5)\ (x - 6)$

$X^2 - 6x - 5x + 30$

$X^2 - 11x + 12$



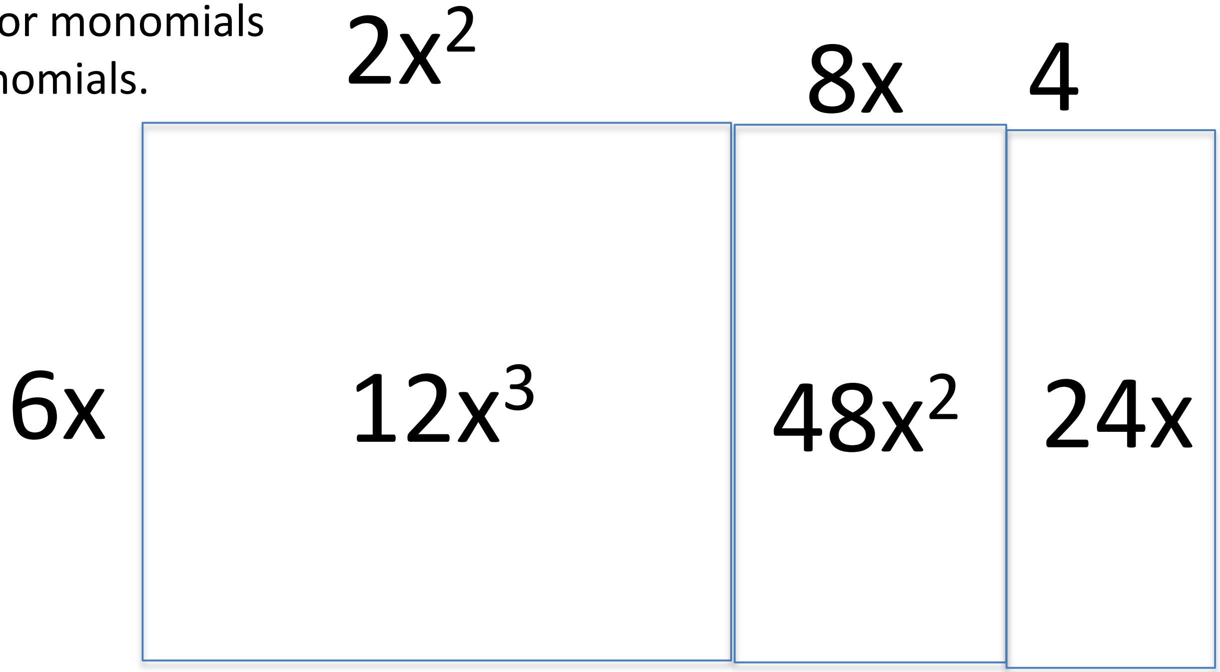


$(x - 2)\ (x + 8)$

$X^2 + 8x - 2x - 16$

$X^2 + 6x + 12$

You can also use the box method for monomials and polynomials.



$(6x)(2x^2 + 8x + 4)$

$12X^3 + 48x^2 + 24x$

You can try these without the box. You can foil, make the box, or do it in your head. Solve on the white board.

Solve on whiteboard.

$$(x - 5)\ (x - 7)$$

$$X^2 - 12x + 35$$

Solve on whiteboard.

$$(x - 6)\ (x - 9)$$

$$X^2 - 15x + 54$$

Solve on whiteboard.

$(x - 4)\ (x + 5)$

$X^2 + x - 20$

This next group has an interesting pattern. See if you can figure it out.

$(x + 11)\ (x - 11)$

$X^2 - \ 121$

$(x + 12)(x - 12)$

$X^2 - 144$

$(x + 13)\ (x - 13)$

$X^2 - 169$

$(x - 14)(x + 14)$

$X^2 - 196$

$(x - 15)(x + 15)$

$X^2 - 225$

$(x + 16)(x - 16)$

$X^2 - 256$

$(x + 17)(x - 17)$

$X^2 - 289$

SEE ANY PATTERNS?

The middle terms cancelled each other out.

In the end, you are left with what is sometimes called, “the difference of two perfect squares.” For example: $x^2 – 16$ can be factored into $(x + 4)\ (x – 4)$

How does this relate to regular numbers? You can use this pattern to multiply numbers in your head.

$(29)(31)$

THINK! 29 is really 30 – 1 and 31 is really 30+ 1

$(30 -1)(30 +1)$

$900 +30 – 30 -1$

899

$(19)(21)$

THINK!

$(20 - 1)(20 + 1)$

$400 + 20 – 20 - 1$

399

$(49)(51)$

$(50 - 1)(50 + 1)$

$2500 + 50 – 50 - 1$

2499

$(18)(22)$

THINK!

$(20-2)(20+2)$

$400 + \cancel{40 - 40} - 4$

396

CAN YOU THINK OF OTHERS?

You could even try numbers where they aren’t the same distance away from 20 or 30:

18 * 21 = (20 -2)(20 +1)
17 * 24 = (20 -3) (20 +4)

FACTORING!

Factoring is breaking down numbers and terms into their simplest parts then looking for things in common. It can help you transform expressions to reveal something new about them. It's similar to writing a linear equation in different ways in order to see slope or intercepts:

$ax + by = c$ $y = mx + b$

For example you can write 6 x 8 as:

6 x 8 = (3 *2) (2 * 2 * 2)

3 * 2 * 2 * 2 * 2 =

6 * 2 * 2 * 2 = 48

You used factoring to find the product.

Another example:

$6 \times 12 = (3 * 2 * 2 * 2 * 3)$

$2^3 * 3^2$

You used factoring to simplify the product.

You can also work it with adding:

8 + 6 = (2 * 4) + (2 * 3)

What do they have in common?

Both expressions are * 2

So, rewrite as: 2 (4 +3) = 14

It’s like the reverse of the distributive property

Example with two terms: $6x^5 + 12y^4$

Rewrite by breaking down… or just think

$(\underline{6x^5}) + (2*\underline{6y^4})$

What do the terms have in common?

6. So we say you can “factor out the six”

And rewrite as: $6(x^5 + 2y^4)$

Try these. Factor out what is in common.

$$12x^2 + 6x$$

$$2x\ (6x + 3)$$

Try these. Factor out what is in common.

$$3x^3 + 6x + 12$$

$$3\,(x^3 + 2x + 4)$$

Try these. Factor out what is in common.

$$4xy^3 + 6xy$$

$$2xy\ (2y^2 + 3)$$

Try these. Factor out what is in common.

$$5x^3y^2 + 10xy$$

$$5xy\ (x^2y + 2)$$